MIN AND MAX VALUES

	Min Value	Max Value
Sin θ	**-1**	**1**
Cos θ		
$\sin^2\theta$	**0**	**1**
$\cos^2\theta$		

Questions on Min and Max Values

Type I

Exp3:- Find the min and max value of $2\sin^2\theta + 3\cos^2\theta$

Sol: $2\sin^2\theta + 3\cos^2\theta = 2\sin^2\theta + 2\cos^2\theta + \cos^2\theta = 2(\sin^2\theta + \cos^2\theta) + \cos^2\theta = 2(1) + \cos^2\theta$

Now since we have converted the given expression into such an exp where we have only 2 and $\cos^2\theta$ since 2 is constant so min and max value of exp depends of min and max value of $\cos^2\theta$. Means value of exp will be min when value of $\cos^2\theta$ is min that is '0'. If we take value of $\cos^2\theta$ = 0 value of exp becomes 2 + 0 = 2 i.e. Min value of exp is 2. ||ly Max value of exp is 2 + 1 = 3 (since max value of $\cos^2\theta$ is 1)

SHORTCUT If we get a question of the type $a\cos^2\theta + b\sin^2\theta$ than min and Max value of exp is

Min value of exp = Min (a, b) **Max value of exp = Max (a, b)**

Type 2

Find the Min value of **$a\sin^2\theta + b\,\mathrm{cosec}^2\theta$**
i.e. 2 Trig ratios where one is reciprocal of other. Like in this case $\mathrm{cosec}^2\theta$ is reciprocal of $\sin^2\theta$.
In these types of questions examiner will always ask for Min value.

Shortcut formula for that is

Min Value of exp = $2\sqrt{ab}$

Proof: $a\sin^2\theta + b\,\mathrm{cosec}^2\theta = \left(\sqrt{a}\sin\theta\right)^2 + \left(\frac{\sqrt{b}}{\sin\theta}\right)^2$

$= \left(\sqrt{a}\sin\theta\right)^2 + \left(\frac{\sqrt{b}}{\sin\theta}\right)^2 - 2\sqrt{a}\sin\theta\frac{\sqrt{b}}{\sin\theta} + 2\sqrt{a}\sin\theta\frac{\sqrt{b}}{\sin\theta}$

$= \left(\sqrt{a}\sin\theta - \frac{\sqrt{b}}{\sin\theta}\right)^2 + 2\sqrt{ab}$ (we know that min value of a exp with sq is '0')

Hence min value of exp = $0 + 2\sqrt{ab} = 2\sqrt{ab}$

<u>Type 3</u> **Find the min and max value of a sin θ + b cosθ**

In these type of question **Min value = -** $\sqrt{a^2 + b^2}$ **and Max Value =** $\sqrt{a^2 + b^2}$